DECISION-MAKING UNDER RISK IN THE OPTIMIZATION OF THE RADIOLOGICAL PROTECTION APPLIED TO DESIGN

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ABSTRACT

When applying optimization of the radiological protection to design, the corresponding data and parameters may present uncertainties. This paper presents a methodology which allows optimization to be kept under the field of decision-making under risk. This methodology involves the use of the Principle of Maximum Entropy, in order to generate a probability distribution from the available information, while the analytical solution may be assessed by applying the Monte Carlo Method.

INTRODUCTION

When applying optimization of protection procedure to design, lack of precise data on the performance of the protection options, uncertainties due to modeling, or the intrinsic uncertainties of random variables, often appears. If the probability distributions of these variables are unknown, it configures a case of decision-making under uncertainty.

The common way of dealing with this situation consists in assigning most likely values to the mentioned data and, consequently, considering it as a deterministic problem. In order to validate this solution, a sensitivity analysis is carried out.

As an alternative to the above mentioned procedure, another methodology is proposed, which allows to consider the situation as a decision-making under risk one. On doing this, information concerning the data may be recalled from experts, and by applying the Principle of Maximum Entropy to this information, its expert-related distribution is determined. Then, the analytical solution may be assessed by applying the Monte Carlo Method.

Optimization of radiation protection taken as a decision-making under risk problem consists in choosing some option a_i from the set of options A, without knowing in advance the exact state, s_i, that nature will adopt from the set of possible states, S. The probabilities of the states of nature may be denoted P_j; therefore, the optimum action a_i^* consists in selecting the protection level i associated with action a_i^*, in such a way that:

$$E(U_i) = \sum_{j=1}^{n} U_{ij} P_j = \max \qquad (1)$$

under the constraints:

$$y_r = f_r(X_1, X_2, \ldots, X_k) \le b_r \qquad r = 1, 2, \ldots, q \qquad (2)$$

$$P[\, y_r = f_r(X_1, X_2, \ldots, X_k) \le \Theta_r \,] \ge D_r \qquad r = q+1, \ldots, R \qquad (3)$$

where Y=f(x) denotes functional dependence, while U_{ij} represents the utility of option i, if state j occurs [1,2], and it depends upon design variables (X) through U=f(X). Among these design variables, collective dose S and costs C should be included, as well as individual dose (H) limitations should be established.

When probabilistic constraints occur, as in expressions (3), only a probability value of attaining a goal, (Dr), may be adopted. If such a goal consists in complying with the individual dose limitation, these limits should be expressed in a probabilistic form [1].

PRINCIPLE OF MAXIMUM ENTROPY OF THE INFORMATION

The probability distribution of any variable may be estimated from information given by experts [3], by applying the Principle of Maximum Entropy (PME). Given the states of nature, s, belonging to an interval R, and being f(s) a probability density function associated to R, a measure of the uncertainty is provided by the entropy of the information, H(f). The PME indicates that, in any case of decision-making under risk, the probability density function which maximizes entropy [4], should be applied.

$$\max \left\{ H(f) \right\} = \max \left\{ -\int_R f(s) \ln\left[f(s) \right] ds \right\} \quad (4)$$

subject to

$$\int_R f(s)\, ds = 1 \; ; \qquad f(s) \geq 0 \quad (5)$$

and any constraint given by the available information, such as:

$$\int_R s\, f(s)\, ds = med; \qquad \int_R (s-med)^2 f(s)\, ds = \sigma^2 \quad (6)$$

where, med and σ^2 represent values assessed by experts. As a final remark, the PME defines an unique probability distribution, which is the most disperse one, compatible with the available information. The above mentioned concept justifies the conservatism of the results obtained from its application.

MONTE CARLO METHOD

This is a numerical technique, which allows the assessment of the probability distribution for a given non-linear function, by simulating random variables. For each one of them, a set of values is generated, with the same distribution corresponding to such a variable. These values are replaced in the mathematical model that characterizes the system, defining its response. The distribution of the dependant variable [5], which conceptually represents a hypothetical sampling of the system performance, may be obtained.

NUMERICAL EXAMPLE

As a demonstration, a simplified version of the optimization technique applied to the shielding thickness (t) of a container used for the transfer of burnt fuel elements at Embalse nuclear power plant is presented. This container is a cylindrical vessel, lead shielded and steel jacketed. Both, lead and steel thicknesses must be the same for the lateral surface, the basement and the top. The corresponding values for the calculation parameters are:

- -Cost of unit of collective dose = 10 000 US$/man Sv
- -Cost of unit mass of lead = 10 US$/kg
- -Annual individual dose limit = 20 mSv/a

The container will be used throughout 2570 operations during 28 years. On performing the operations, workers identified as A, B, C, D and E will be exposed to radiation, as described below: Workers A and B will carry out tasks 25A and 25B, in an alternate chronogram. Workers C, D and E will deal with tasks 27, 28 y 30 in the same way. As a consequence of this working scheme, workers A and B will receive equal doses, as workers C, D and E will do [6].

Each task is characterized by the time required for its fulfillment (T), and by the distance between the workers and the container (WCD). Both values were obtained from the designers. The values were asked as a mean (MED), as a minimum (MIN) and as a maximum value (MAX). These values are presented at Table 1.

TASK	T (min.)					WCD (m)				
	MIN.	MED.	MAX.	c	μ	MIN.	MED.	MAX.	c	μ
25A	6.0	10.0	30.0	1.079	-0.246	0.2	1.0	5.0	1.577	-1.230
25B	6.0	10.0	30.0	1.079	-0.246	2.5	3.0	3.5	1.0	1.0
27	2.0	3.0	6.0	5.569	-0.898	0.2	1.0	1.5	0.278	1.001
28	2.0	5.0	7.0	0.062	0.246	0.5	1.0	1.3	0.196	1.950
30	0.7	1.0	3.0	33.97	-3.321	0.2	0.5	0.6	0.042	8.984

Table 1. Estimated time and distance for each task, and determined by the Principle of Maximun Entropy.

Equivalent doses (DC) for different distances and shielding thicknesses were calculated by using MERCURE-4 software [7,8]. Based upon experts opinion, it was estimated that the actual doses should be normally distributed with a mean value of 0.8 DC and a variance of 0.2 DC .

The mean values in Table 1 and the estimated doses (DC) were used in order to obtain the solution by applying the cost-benefit technique. The corresponding results are presented in Table 2.

t (cm Pb)	C (US$)	S (Sv)	U=C+αS (US$)	H(AB) (mSv/year)	H(CDE) (mSv/year)
7	38230	3.9307	77537	37.24	21.54
8	44590	2.0706	65296	19.57	11.33
9	51190	1.1503	62693*	10.81	6.33
10	58040	0.6193	64233	5.86	3.38
11	65130	0.3406	68536	3.23	1.85

Table 2. Results obtained by applying cost-benefit technique.

In order to obtain the solution by the proposed methodology, the principle of Maximum Entropy was applied to the available data. The corresponding distribution for each variable was assessed by using expressions (4) and (5), and constraint (6) as:

$$\int_{MIN.}^{MAX.} s\ f(s)\ ds = MED \qquad (7)$$

Proper solutions were determined [4] by applying calculus of variations, resulting in $f(s)=ce^{\mu s}$, while constants c and μ were calculated by using constraints (5) and (7), and are presented in Table 1. The analytical solution was obtained by applying the Monte Carlo technique, which allowed the determination of the distribution of individual and collective doses (Table 3).

t (cm Pb)	C (US$)	E(U) (US$)	E[H(AB)] (mSv/y)	E[H(CDE)] (mSv/y)	P(1)	P(2)
7	38230	81525	46.27	30.49	0.2552	0.1439
8	44590	67456	24.42	16.11	0.5082	0.7892
9	51190	63929	13.58	9.02	0.7918	0.9943
10	58040	64712	7.11	4.75	0.9695	1.0
11	65130	68853	3.99	2.62	0.9968	1.0

Table 3. Results obtained by applying decision-making under risk. P(1) denotes $P[H(AB) \leq 20mSv/y]$; while $P(2)=P[H(CDE) \leq 20mSv/y]$.

According to the cost-benefit technique, the optimized thickness should be 9 cm lead, with a maximum individual dose of 11 mSv/a. The corresponding results for the proposed methodology should be an equal thickness, but a probability P(1) of only 79 % for individual limitation compliance should be accepted.

Cost-benefit techniques require a sensitivity analysis in order to evaluate the stability of its solutions. This analysis presents some problems, such as the lack of a proper methodology of input data selection or that, under certain circumstances, different results are obtained when changing the input data.

On the contrary, the proposed methodology emphasizes the probabilistic features found in most cases in optimization of protection, dealing with the available information accordingly with decision analysis axioms.

CONCLUSIONS

The proposed methodology has demonstrated to be a useful tool for decision-making applied to the optimization of radiation protection under uncertainties. These techniques are adequate in dealing with any kind of random variable, delivering non-contradictory results, with a high degree of coherency with the analyzed situation.

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